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which in the course of time become central axes, let O_1 be a point fixed in space and $\dot{O}$ the velocity of that point of the body which at the time t coincides with O_1. Then the required equation is

$$R = O_1 + a\boldsymbol{w} + \frac{1}{\boldsymbol{w}^2}\boldsymbol{w} \wedge \dot{O}. \tag{14}$$

19. By (14), the velocity of any point R of the central axis is

$$\dot{R} = a\dot{\boldsymbol{w}} + \frac{d}{dt}\left(\frac{1}{\boldsymbol{w}^2}\boldsymbol{w} \wedge \dot{O}\right).$$

The ruled surface (14) is developable, *i. e.*, the motion of the central axis is a pure rotation, if the velocity $\dot{R}$ is parallel to the plane of the vectors $\boldsymbol{w}$ and $\dot{\boldsymbol{w}}$, *i. e.*, if $\dot{R} \times \boldsymbol{w} \wedge \dot{\boldsymbol{w}} = 0$. The condition for the surface (14) to be developable is therefore

$$\left(\frac{d}{dt}\frac{1}{\boldsymbol{w}^2}\boldsymbol{w} \wedge \dot{O}\right) \times \boldsymbol{w} \wedge \dot{\boldsymbol{w}} = 0,$$

i. e.,

$$\left(-\frac{2\boldsymbol{w} \times \dot{\boldsymbol{w}}}{\boldsymbol{w}^2}\boldsymbol{w} \wedge \dot{O} + \dot{\boldsymbol{w}} \wedge \dot{O} + \boldsymbol{w} \wedge \ddot{O}\right) \times \boldsymbol{w} \wedge \dot{\boldsymbol{w}} = 0. \tag{15}$$

20. This equation contains $\ddot{O}$ only in the combination $\ddot{O} \times (\boldsymbol{w} \wedge \dot{\boldsymbol{w}}) \wedge \boldsymbol{w}$; *i. e.*, if $\ddot{O}$ be resolved along the rectangular vectors $\boldsymbol{w}$, $\boldsymbol{w} \wedge \dot{\boldsymbol{w}}$, and $\boldsymbol{u} = \boldsymbol{w} \wedge (\boldsymbol{w} \wedge \dot{\boldsymbol{w}})$, the equation contains only the component of $\ddot{O}$ along $\boldsymbol{u}$.

This fact has an interesting dynamical interpretation. Let O be the centroid, and let $\boldsymbol{R}$, $\boldsymbol{H}$ be the vectors of the resultant force and couple for O. Then our result means that, if the surface of the screw axes is to be developable, $\boldsymbol{R}$ must have a fixed projection on $\boldsymbol{u}$; in other words, all values of $\boldsymbol{R}$ that give a developable surface are such that, when drawn from O, their extremities lie in a plane parallel to $\boldsymbol{w}$ and $\boldsymbol{w} \wedge \dot{\boldsymbol{w}}$.

BOOK REVIEWS.

SEND ALL COMMUNICATIONS TO W. H. BUSSEY, University of Minnesota.

Contributions to the Founding of the Theory of Transfinite Numbers. By GEORG CANTOR. Translated and provided with an introduction and notes by PHILIP E. B. JOURDAIN. The Open Court Publishing Company, Chicago, 1915. ix+211 pages.

The main purpose of this little volume, the first in the Open Court Series of Classics of Science and Philosophy, is to make accessible in English the two final papers of Georg Cantor on the theory of transfinite numbers, which embody the culmination of his researches on this subject, and which appeared in 1895 and 1897 in the *Mathematische Annalen* (Vols. 46 and 49). In order to give a proper setting to these two papers, the translator precedes them by an introduction of about 80 pages, in which he sketches briefly the progress of ideas which lead up to these researches and results. A brief note collecting some of the chief developments which have been made in this theory since that time, brings the volume to a close.

In the introduction, we find some brief indications of the genesis of the modern conception of function, out of the consideration of the problem of the vibrating string and the theory of Fourier series. This is followed by a sketch of Weierstrass's theory of irrational numbers, in so far as Weierstrass probably exerted a most potent influence on Cantor. The main section of the introduction, however, is devoted to a résumé of the chief results obtained by Cantor in his researches preceding the papers translated. Beginning with his investigations in the theory of Fourier series, we are shown how Cantor was led to a closer study of point sets, on the one hand in the direction of their " Maechtigkeit " or power, and on the other to the consideration of their derivatives of various orders, the latter culminating in the notion of transfinite ordinal numbers.

The desirability of such an historical introduction is of course apparent. Aside from its value in preparing one for the papers which form the central portion of the book, it is always an interesting matter to trace the development of an idea, to go behind the scenes, so to speak, and see how a bit of investigation gradually assumes a final form. It is therefore very much to be regretted that our author in his attempts to be brief is not always perfectly clear. For instance his exposition of Weierstrass's theory of irrational numbers, on pages 18 and 19, would appeal to one who is not thoroughly conversant with the theory in question as a hopeless conglomeration of aggregates, sums, numerical quantities, and other elements. A simple illustration would probably have served the purpose much better than this attempt at generality without definiteness. Later on, in Section VII, page 52 and following, in the résumé of Cantor's 1883 Grundlagen, much would have been gained in clearness if the author had followed Cantor's original article a little more closely. In particular, the meaning of the word "Anzahl," which is translated enumeral, and seems to lead to confusion in the mind of the author, would have been perfectly clear at the outset as referring particularly to arrangement, rather than to " Maechtigkeit " or power. As it is, only after it has been used repeatedly and frequently do we really get any inkling as to the meaning to be assigned to the term enumeral. Of other sections which might well be improved, we might note the discussion of DuBois Reymond's work on point sets, on page 34; and the proof that the totality of real numbers is not enumerable on page 39.

To give the contents of the papers translated is hardly within the province of this review. Briefly, they contain the developments of the notion of cardinal number, that is, the power of a set, and the laws governing the usual algebraic processes as applied to them. A similar treatment is given for the ordinal numbers. The second of the two papers concerns itself mainly with well-ordered sets. The translation, considered as a whole, is fairly well done. Occasional passages to be sure are reminiscent of the German idiom, and perhaps too literal. For instance, the statement of Theorem H on page 147 is ambiguous as a result of sticking too closely to the German text.

In its general makeup and appearance, we think the book commendable. As a personal opinion, we feel that it would have tended towards clarity if the translator had followed Cantor in italicizing the theorems. Further, we find the printing of the page numbers of the original papers in the text in bold-faced type somewhat disconcerting. They might well have been relegated to the margin and a smaller size of type used. Of misprints, which are liable to cause ambiguity, we note: on page 147, line 14, D ought to be replaced by G; and in line 21, F ought to be replaced by f.

As a contribution to the history of transfinite numbers, and in making the main two papers on the foundations of these numbers accessible in English, we think this little volume is worthy of consideration. As an introduction to the subject, however, it cannot be unqualifiedly recommended.

T. H. HILDEBRANDT.

THE UNIVERSITY OF MICHIGAN.

Solid Geometry. By SOPHIA FOSTER RICHARDSON. Ginn and Co., Boston, 1914. v + 209 pages.

In applying to Professor Richardson's solid geometry the severe test of class-room use, it has been evident that the book possesses many excellent qualities. The use of a single letter to denote a line or a plane and the names skew lines, pencil of lines, pencil of planes, bundle of lines, bundle of planes have tended to make the work concise and the class demonstrations easy to follow. The first chapter contains more theorems than is usual in books on solid geometry. It provides a good drill in the more simple propositions that fix the concepts of perpendicularity and parallelism in space. A feature of interest to the student is the illustration of the principle of duality in several theorems differing only in the interchange of the words line and plane. Throughout the book there is an abundance of valuable and interesting exercises. However in many of the demonstrations a part of the work might well have been left for the student to supply.

The typographical errors are few, and in no instances misleading. The treatment of the incommensurable cases based on the Dedekind-Cantor theory of irrational numbers, presented in the appendix, is doubtless rigorous, but its advantages over the more ordinary methods have not been evident, since otherwise the work could be made much shorter and as rigorous as can be appreciated by first year college students. The propositions leading up to the determination of the surface and the volume of a sphere seem unnecessarily long and so involved that it becomes an arduous task even to quote them.

The formula for the volume of any parallelopiped is developed in two theorems, thus eliminating the celebrated "devil's coffin." This ruthless vandalism is regretted perhaps more by the expectant freshman, who has heard of its fame, than by the instructor, who has become calloused to its fearful and wonderful construction.

Taking everything into consideration it would seem that Professor Richardson